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$J/\Psi \to \phi\pi\pi(K\overline{K})$ decays, chiral dynamics and OZI violation

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We have studied the invariant mass distributions of the $\pi\pi$ and $K\overline{K}$ systems for invariant masses up to 1.2 GeV from the $J/\Psi \to \phi\,\pi\pi(K\overline{K})$ decays. The approach exploits the connection between these processes and the $\pi\pi$ and $K\overline{K}$ strange and non-strange scalar form factors by considering the ϕ meson as a spectator. The calculated scalar form factors are then matched with the ones from next-to-leading order chiral perturbation theory, including the calculation of the the $K\overline{K}$ scalar form factors. Final state interactions in the $J/\Psi \to \phi\pi\pi(K\overline{K})$ processes are taken into account as rescattering effects in the system of the two pseudoscalar mesons. A very good agreement with the experimental data from DM2 and MARK-III is achieved. Furthermore, making use of SU(3) symmetry, the S-wave contribution to the $\pi^+\pi^-$ event distribution in the $J/\Psi \to \omega\pi^+\pi^-$ reaction is also given and the data up to energies of about 0.7 GeV are reproduced. These decays of the J/Ψ to a vector and a pair of pseudoscalars turn out to be very sensitive to OZI violating physics which we parameterize in terms of a direct OZI violation parameter and the chiral perturbation theory low energy constants L_4^r and L_6^r. These constants all come out very different from zero, lending further credit to the statement that the OZI rule is subjected to large corrections in the scalar 0^{++} channel.

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I. INTRODUCTION

The decays of the J/Ψ into a ϕ meson and Goldstone boson pair ($\pi\pi$ or $K\overline{K}$) can be used to investigate the dynamics of the interacting pseudoscalars. In particular, it was argued in Ref. [1] that these data together with data from pion–pion scattering (and from others sources) force the $f_0(980)$ to have a pole structure different to the one required by a $K\overline{K}$ molecule [2]. This interpretation has been challenged, e.g. in the Jülich meson–exchange model where the $f_0(980)$ emerges [3] as a $K\overline{K}$ bound state. Furthermore, in this reference the authors are also able to reproduce the data associated with the previous J/Ψ decays within the same formalism than the one employed in Ref. [1], but making use of their own strong amplitudes. On the other hand, as will be the topic of this investigation, these data can be used to study the violation of the Okubo–Zweig–Iizuka (OZI) rule in the scalar (0^{++}) channel. This rule is only well founded in the large N_c limit of QCD, with N_c the number of colors, since OZI violating processes are described by suppressed non–planar graphs [5]. Still, on a purely phenomenological level this rule works astonishingly well, with the exception of the scalar channel, as argued e.g. in Refs. [6], [7], [8]. To be more precise, the decay $J/\Psi \to \phi M\bar{M}$ (where $M\bar{M}$ denotes the pseudoscalar meson pair) is OZI suppressed to leading order, cf. Fig.1a, but has an additional doubly OZI suppressed contribution depicted in Fig.1b. In our approach, both these pieces are taken into account. In fact, it will turn out that the second contribution can not be neglected if one wants to achieve an accurate description of the data. On the other hand, it is mandatory to have a very precise description of the final state interaction in the coupled $\pi\pi/K\overline{K}$ system (as indicated by the shaded blob in Fig.1a) before one can ask such detailed questions. As can be seen from Fig.1, the crucial ingredient in the reaction at hand are the expectation values of the scalar–isoscalar condensates in the pion and the kaon, i.e the so-called scalar form factors. These can be calculated at low energies in chiral perturbation theory (CHPT), which is the effective field theory of the Standard Model. In our case, the dimeson system can have energies up to 2 GeV and we thus employ unitarity constraints to get a precise description of these scalar form factors also at higher energies, demanding furthermore matching to the CHPT expressions in the low energy domain. Because of this matching procedure, the large N_c suppressed low energy constants L_4^r and L_6^r of the next–to–leading order effective chiral Lagrangian can be determined in the process we are considering. It has been argued before that so far no direct determinations but rather large N_c inspired estimates have been done, see e.g. Refs. [7], [9], with the exception of more recent work presented in Refs. [7,10,11]. Nevertheless, as we will discuss in much more detail below, a rather definite determination of L_4^r can be obtained by considering $\mathcal{O}(p^6)$ CHPT results [12,13].

To be more specific, to address the problem of the final state interactions in the coupled $\pi\pi$-$K\overline{K}$ system, we make use of the results obtained in Ref. [14]. In this paper it was clearly established that the scattering data of the 0^{++} $I=0,1$ (I denotes the total isospin of the dimeson system) sectors up to centre–of-mass energies of 1.2 GeV are a reflection of the strong rescattering effects between the lightest pseudoscalars ($\pi\pi$, $K\overline{K}$ for $I=0$ and $\pi\eta$ and $K\overline{K}$ for $I=1$). The approach was based on Bethe-Salpeter equations using the lowest order CHPT amplitudes [15,16,9] as the driving potential. The fact that one can generate the resonance states for those channels via loop physics, i.e. rescattering, is a clear signal of the large deviations from OZI rule in the 0^{++} sector, see also Refs. [7,6]. Such a mechanism has been advocated since long, for a pedagogic discussion see Ref. [17]. On the other hand, it is well known that there is an on–going controversy concerning the nature of the scalar resonances $f_0(980)$ and $a_0(980)$. This controversy originates from the observation that there are several different models to deal with the $I=0,1$ scalar sector, all of them reproducing the scattering data up to some extend, but with different conclusions with respect to the origin of the underlying dynamics. In particular, in Refs. [18–20] these resonances are considered of preexisting origin while in Ref. [2] they appear as meson–meson resonances originated by a potential. Also in Ref. [21] it is advocated for the solution that the $a_0(980)$, $f_0(980)$ are exotic resonances, that is, not simply $q\bar{q}$, while the preexisting $q\bar{q}$ scalar nonet should be heavier, around 1.4 GeV or so. Other interesting approaches to this problem are the relativistic quark model with an instanton induced interaction of the Bonn group [22], the Jülich meson–exchange approach [3] or the use of QCD sum rules [23]. With respect to this controversy, the contribution of the work in Ref. [14] is very valuable since, at least, the infinite series of diagrams there considered should appear in the whole S–wave partial wave amplitudes calculated to all orders in CHPT. The conclusions of Ref. [14] where generalized in Ref. [24]. In that paper, the most general structure of a partial wave amplitude when the unphysical cuts are neglected was established. In particular, in this paper explicit s-channel resonance exchanges were included together with the lowest order CHPT contribution and the whole $SU(3)$ connected scalar sector with $I=0,1/2,1$ was studied. In particular, it was shown that the amplitudes of Ref. [14] appear as a particular case when removing the explicit tree level resonance contributions. It was observed that the lightest 0^{++} nonet is of dynamical origin, i.e. made up of meson–meson resonances, and is formed by the $\sigma(500)$, κ, $a_0(980)$ and a strong contribution to the physical $f_0(980)$. On the other hand, the preexisting scalar nonet would be made up by an octet around 1.4 GeV and a singlet contributing to the physical $f_0(980)$ resonance. With respect to this last point, as discussed in Ref. [24], the inclusion of a preexisting contribution to the $f_0(980)$ was

considered in order to be able to reproduce the data on the inelastic $\pi\pi \to K\overline{K}$ cross section[1] when including also the $\eta\eta$ channel. However, if this channel is not considered, one can reproduce the strong scattering data, including also the previous experiments on the inelastic $\pi\pi \to K\overline{K}$ cross section, without including such preexisting contribution. Finally, in Ref. [24] the contribution in the physical region of the unphysical cut contributions were estimated up to $\sqrt{s} \approx 800$ MeV to be just a few per cents making use of the results of Ref. [26], which apply below that energy. We will use the formalism of Ref. [14] whose partial wave amplitudes have been also tested in many other reactions. As pointed out in Ref. [1], to obtain a consistent picture of the scalar sector, one also has to study other reactions in which the 0^{++} amplitudes have a possible large influence via final state interactions. In this way one can complement the deficient information coming from the direct strong S–wave scattering data and distinguish between available models. In Ref. [27] all the whole set of photon fusion reactions $\gamma\gamma \to \pi^0\pi^0$, $\pi^+\pi^-$, K^+K^-, $K^0\overline{K}^0$ and $\pi^0\eta$ were reproduced in an unified way from threshold up to $\sqrt{s} \approx 1.4$ GeV making use of Ref. [14] to take care of the final state interactions. The free parameters present in Ref. [27] were fixed by their values from the PDG [25]. In Ref. [28], making use of the formalism set up in Ref. [29] to study the still unmeasured $\phi \to \gamma K^0\overline{K}^0$ decay, the reactions $\phi \to \gamma\pi^0\pi^0$, $\gamma\pi^+\pi^-$ and $\gamma\pi^0\eta$ were predicted. These predictions were nicely confirmed, almost simultaneously, by a recent experiment in Novosibirsk [30]. In this manuscript, we will consider yet another applications of the strong amplitudes calculated in Ref. [14] by studying the $J/\Psi \to \pi\pi(K\overline{K})$ decays. In this way, the present study together with the whole set of works [14,27,29,28] offer a unique theoretical approach to the scalar sector able to discuss all these reactions in an unified way. This is achieved without including new elements ad hoc for each reaction, because all these processes are related by the use of an effective theory description that combines CHPT and unitarity constraints.

Our manuscript is organized as follows. In section II we develop and justify a simple phenomenological model for the transition of the J/Ψ into the ϕ meson and a pseudoscalar meson pair. This model is generalized in section IV by making use of flavor SU(3) symmetry and then applied to the ω case. Section III is devoted to the calculation of the scalar pion and kaon form factors for the non–strange as well as the strange scalar density to one loop. Then we employ the unitarization procedure discussed before to obtain a description of these quantities up to energies of 1.2 GeV. We also perform the matching of these expressions to the one loop CHPT ones to be consistent with the constraints of the broken chiral symmetry of QCD at energies below approximately 0.5 GeV. The results are presented and discussed in section IV where we also generalize section II. Our conclusions and outlook are given in section V.

II. MODELING $J/\Psi \to \phi\pi\pi$, $\phi K\overline{K}$ DECAYS

We will calculate the S-wave contribution to the invariant mass distributions of the $\pi\pi$ and $K\overline{K}$ systems in the $J/\Psi \to \phi\pi\pi(K\overline{K})$ decays. Taking care of the final state interactions of three particles can be simplified to a large extend if only two of the final state particles undergo strong interactions, and the third is merely a spectator. We will assume that this is the case in our present problem and we will take the ϕ as the spectator. This is certainly very well sounded for the $\phi\pi\pi$ system since the $\phi\pi$ interaction is very weak as required by the OZI [4] rule. On the other hand, the situation is not so clear with respect the ϕ and the kaons. Nevertheless, at the energies in which the kaons become important, above the $K\overline{K}$ threshold, the experimental mass distribution is completely dominated by the $f_0(980)$ resonance and this state is a two body effect emerging from the coupled $\pi\pi$ and $K\overline{K}$ systems, as discussed already in the introduction. Since we are only considering a small range of energies above the $K\overline{K}$ threshold, this approximation should be justified.

We therefore describe the transition from the J/Ψ to the ϕ+2 Goldstone bosons system by an effective Lagrangian based on the following phenomenological arguments: 1) The already discussed spectator role of the ϕ resonance and 2) the $\pi\pi$ and $K\overline{K}$ invariant event distributions, which will be shown later, seem to be clearly dominated by the S-wave contribution, although these experimental data have not yet been subjected to a partial-wave analysis. These *experimental* facts, together with Lorentz invariance, can be easily incorporated in the formalism just by writing the interaction vertex of the J/Ψ resonance with the ϕ meson and some scalar source S with vacuum quantum numbers, $J^{PC} = 0^{++}$ with spin J, parity P and charge conjugation C, as:

$$g\, \Psi_\mu \phi^\mu S \tag{1}$$

[1]In the last edition of the PDG tables [25] it is argued that, possibly, the previous experiments have a much larger uncertainty than previously given in the corresponding publications.

with g a real coupling constant. We briefly discuss why other possible structures involving derivatives on the various fields should be suppressed. The J/Ψ is very heavy and thus can be considered a static source. Derivatives acting on the ϕ and the scalar source S can be combined to the invariant structure $\Psi^\mu(\partial_\nu\phi_\mu - \partial_\mu\phi_\nu)\partial^\nu S$. This leads to a vertex of the form $\epsilon^\mu_\Psi \epsilon^\phi_\mu \, p_\phi \cdot (q_1+q_2)$, with $q_{1,2}$ the momenta of the two Goldstone bosons and p_ϕ the momentum of the ϕ meson. However, due to momentum conservation, we have $p_\phi \cdot (q_1+q_2) = (M_\Psi^2 - M_\phi^2 - s)/2$, with $s = (q_1+q_2)^2$ the total two Goldstone boson energy squared. Due to the large value of the J/Ψ mass, this combination of momenta is essentially constant for the Goldstone boson energies considered here, $\sqrt{s} \leq 1.2\,\mathrm{GeV}$. Such terms become more important at higher di–pion (kaon) energies. Therefore, we can generically write such type of higher order corrections to Eq.(1) in the form

$$\epsilon^\mu_\Psi \epsilon^\phi_\mu \, f\left((q_1+q_2)^2, p_\phi\cdot(q_1+q_2)\right) \ , \tag{2}$$

where the function $f(\ldots)$ essentially only depends on the first argument. Such terms that depend on $(q_1+q_2)^2$ can be derived from an interaction of the type $\Psi^\mu\phi_\mu\partial_\nu\partial^\nu S$. Such structures lead to a weak s–dependence of the constant g and/or the parameter λ_ϕ defined below. We have checked that such (weak) energy dependencies do not change any of the conclusions obtained when treating g and λ_ϕ as energy independent. Another possible higher order term of the form $\Psi_\mu\phi_\nu\partial^\mu\partial^\nu S$ giving rise to the vertex $\epsilon^\mu_\Psi\epsilon^\phi_\nu\,(q_1+q_2)_\mu(q_1+q_2)^\nu$. Such couplings can also have an S–wave contribution, which can be obtained by properly summing over the pertinent polarization vectors. Again, due to the large mass of the J/Ψ, such terms are only weakly s dependent and can be treated along the lines outlined before. More complicated structures can always be brought into some linear combination of the ones just discussed or have no S–wave component. These considerations not only show that our ansatz Eq.(1) is quite sensible in the energy range considered here but also that corrections to it can be worked out consistently.

From the OZI rule, which can also be seen as a result of the large N_c expansion of QCD [5], and the experimental absence of any clue indicating a non negligible interaction between the ϕ and the pions, one should expect that this scalar source S would be simply made of strange quarks, i.e. $S \sim \bar{s}s$. However, it is known that the ϕ also decays into non–strange mesons and furthermore, there are strong arguments to believe that large violations of the OZI rule (and of the large N_c limit of QCD) are manifest in the 0^{++} sector [7,6,24,14], as discussed in the introduction. As a result, we will consider a more general scalar source S, that also has a contribution of the form $\lambda_\phi\bar{n}n$, where

$$\bar{n}n = \frac{1}{\sqrt{2}}\,(\bar{u}u + \bar{d}d) \tag{3}$$

parameterizes the contribution from the non–strange quarks and λ_ϕ is just a constant measuring the relative strength of this contribution with respect to the strangeness component $\bar{s}s$. Already at this point we stress that the choice $\lambda_\phi \neq 0$ will be justified a posteriori by the results presented below. Therefore, we use

$$S = \bar{s}s + \lambda_\phi\,\bar{n}n \tag{4}$$

Consequently, it follows from Eqs.(1) and (4) that the transition matrix element for the process $J/\Psi \to \phi\, M\bar{M}$ is given by:

$$T = \epsilon(\Psi;\,\rho)_\mu \epsilon(\phi;\,\rho')^\mu \,\langle 0|\,(\bar{s}s + \lambda_\phi\,\bar{n}n)\,|M\bar{M}\rangle^* \tag{5}$$

where $|0\rangle$ is the vacuum state, $\epsilon(\Psi;\,\rho)$ is the polarization four–vector of the J/Ψ resonance with polarization ρ and analogously $\epsilon(\Psi;\,\rho')$ is the polarization four–vector of the ϕ resonance, and the * denotes complex conjugation. Note that in this equation we are implicitly assuming that the ϕ is a spectator as discussed before. The scalar source S couples to the two meson system, in which the rescattering (final state interactions) appear. The anatomy of our model is depicted in Fig.2. As discussed below, invoking SU(3) symmetry, we will also apply this approach to the S–wave contribution of the $J/\Psi \to \omega\pi\pi$ decay to further constrain the description of the measured event distributions.

III. COUPLED CHANNEL PION AND KAON SCALAR FORM FACTORS

As a consequence of Eq.(5), our problem is reduced to calculate the matrix elements $\langle 0|\bar{s}s|M\bar{M}\rangle$ and $\langle 0|\bar{n}n|M\bar{M}\rangle$, which correspond to the strange and non-strange isospin zero ($I=0$) scalar form factors.

A. Definition of the scalar form factors

One can define an extended QCD Lagrangian allowing for the presence of external sources. In this way the identification of matrix elements of quark currents can be done easily. For instance, a scalar source can be added simply as:

$$
-\,\bar{q}\,\Sigma\, q\ , \tag{6}
$$

where q embodies the three light quarks, u, d and s. The QCD current quark mass term can be obtained from such a scalar source by setting,

$$
\Sigma = \mathrm{diag}(m_u, m_d, m_s)\ . \tag{7}
$$

This is the standard method of treating explicit chiral symmetry breaking in CHPT (or any similar effective field theory). Consequently, we can work out the scalar quark–antiquark operators,

$$
\begin{aligned}
\bar{u}u &= -\frac{\partial \mathcal{L}_{\mathrm{QCD}}}{\partial \Sigma_{11}} = -\frac{\partial \mathcal{L}_{\mathrm{QCD}}}{\partial m_u}\ ,\\
\bar{d}d &= -\frac{\partial \mathcal{L}_{\mathrm{QCD}}}{\partial \Sigma_{22}} = -\frac{\partial \mathcal{L}_{\mathrm{QCD}}}{\partial m_d}\ ,\\
\bar{s}s &= -\frac{\partial \mathcal{L}_{\mathrm{QCD}}}{\partial \Sigma_{33}} = -\frac{\partial \mathcal{L}_{\mathrm{QCD}}}{\partial m_s}\ .
\end{aligned} \tag{8}
$$

One can include in the same way as in the QCD Lagrangian the external sources in the effective CHPT Lagrangian [9] simply based on symmetry arguments. In the lowest order chiral effective Lagrangian, $\mathcal{L}_2$, the scalar source appears in the mass term

$$
\mathcal{L}_2^{\mathrm{mass}} = \frac{1}{4} f^2 \langle U^\dagger \chi + \chi^\dagger U \rangle\ , \tag{9}
$$

with f the meson decay constant (in the chiral limit), $\chi \equiv 2B_0\Sigma$ and B_0 is a constant not fixed by symmetry. This constant parameterizes the strength of the quark–antiquark condensation in the non–perturbative vacuum, $B_0 = |\langle 0|\bar{q}q|0\rangle|/f^2$. The trace in flavor space is denoted by $\langle \ldots \rangle$. The octet of Goldstone bosons is collected in the matrix–valued unimodular field $U(x)$,

$$
U = \exp\left(\frac{i\sqrt{2}}{f}\Phi\right) \tag{10}
$$

with

$$
\Phi = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{bmatrix}\ . \tag{11}
$$

It is then straightforward to work out the scalar–isoscalar quark-antiquark operators from the effective Lagrangian,

$$
\begin{aligned}
\bar{u}u &= -\frac{\partial \mathcal{L}_2}{\partial \Sigma_{11}} = -f^2 B_0 \left[1 - \frac{1}{f^2}\left(\pi^+\pi^- + K^+K^- + \frac{(\pi^0)^2}{2} + \frac{\eta_8^2}{6} + \frac{\pi^0\eta_8}{\sqrt{3}}\right) + ...\right]\\
\bar{d}d &= -\frac{\partial \mathcal{L}_2}{\partial \Sigma_{22}} = -f^2 B_0 \left[1 - \frac{1}{f^2}\left(\pi^+\pi^- + K^0\overline{K}^0 + \frac{(\pi^0)^2}{2} + \frac{\eta_8^2}{6} - \frac{\pi^0\eta_8}{\sqrt{3}}\right) + ...\right]\\
\bar{s}s &= -\frac{\partial \mathcal{L}_2}{\partial \Sigma_{33}} = -f^2 B_0 \left[1 - \frac{1}{f^2}\left(K^+K^- + K^0\overline{K}^0 + \frac{2}{3}\eta_8^2\right) + ...\right]
\end{aligned} \tag{12}
$$

where the ellipsis denotes terms of higher order in the meson fields not needed here. From the last of these equations one concludes that the strangeness component of the pion should be small since it only comes in at higher orders. From this representation of the scalar operators, one can deduce the pertinent expressions for the scalar form factors. For our purposes, it is sufficient to consider pure isospin zero ($I = 0$) states formed from a pion or kaon–anti-kaon pair, i.e.

$$
\begin{aligned}
|\pi\pi\rangle &= \frac{1}{\sqrt{6}}|\pi^+\pi^- + \pi^-\pi^+ + \pi^0\pi^0\rangle \ , \\
|K\bar{K}\rangle &= \frac{1}{\sqrt{2}}|K^+K^- + K^0\overline{K}^0\rangle \ .
\end{aligned} \tag{13}
$$

Note the extra factor $1/\sqrt{2}$ in the definition of the $I=0$ $|\pi\pi\rangle$ state, it is introduced to take care that in the isospin basis states the pions behaves as identical particles. Combining this with Eq.(12), one can easily calculate the lowest order (tree level) CHPT results (remember the normalization of the non–strange quark operator given in Eq.(3)),

$$
\begin{aligned}
\langle 0|\bar{n}n|\pi\pi\rangle &= \sqrt{3}\, B_0 \ , \\
\langle 0|\bar{n}n|K\overline{K}\rangle &= B_0 \ , \\
\langle 0|\bar{s}s|\pi\pi\rangle &= 0 \ , \\
\langle 0|\bar{s}s|K\overline{K}\rangle &= \sqrt{2}\, B_0 \ .
\end{aligned} \tag{14}
$$

As anticipated, to leading order the two–pion system has no strangeness component. To all orders, these matrix elements are given in terms of four *scalar* form factors,[2]

$$
\begin{aligned}
\langle 0|\bar{n}n|\pi\pi\rangle &= \sqrt{2}\, B_0\, \Gamma_1^n(s) \ , \\
\langle 0|\bar{n}n|K\overline{K}\rangle &= \sqrt{2}\, B_0\, \Gamma_2^n(s) \ , \\
\langle 0|\bar{s}s|\pi\pi\rangle &= \sqrt{2} B_0\, \Gamma_1^s(s) \ , \\
\langle 0|\bar{s}s|K\overline{K}\rangle &= \sqrt{2}\, B_0\, \Gamma_2^s(s) \ .
\end{aligned} \tag{15}
$$

Here, the following notation is employed. The superscript s/n refers to the strange/non–strange quark operator whereas the subscript $1,2$ denotes pions and kaons, respectively. In the following we will remove from Eqs.(14,15) the overall factor $\sqrt{2}B_0$, since the experimental data on the $J/\Psi \to \phi M\bar{M}$ decays are not normalized.

B. Next-to-leading order pion and kaon scalar form factors

The pion scalar form factors $\Gamma_1^n(s)$ and $\Gamma_1^s(s)$ were calculated in Ref. [9] up to one loop in CHPT. Since they were not explicitly given in Ref. [9], we give here the pertinent expressions:

$$
\begin{aligned}
\Gamma_1^n(s) &= \sqrt{\frac{3}{2}}\left\{1 + \mu_\pi - \frac{1}{3}\mu_\eta + \frac{16 m_\pi^2}{f^2}\left(2L_8^r - L_5^r\right) + 8(2L_6^r - L_4^r)\frac{2m_K^2 + 3m_\pi^2}{f^2} + f(s) + \frac{2}{3}\,\widetilde{f}(s)\right\} \ , \\
\Gamma_1^s(s) &= (2L_6^r - L_4^r)\frac{8\sqrt{3}\, m_\pi^2}{f^2} + \frac{1}{\sqrt{3}}\widetilde{f}(s) \ ,
\end{aligned} \tag{16}
$$

with $f(s)$ and $\widetilde{f}(s)$ given by

$$
\begin{aligned}
f(s) &= \frac{2s - m_\pi^2}{2f^2}\bar{J}_{\pi\pi}(s) - \frac{s}{4f^2}\bar{J}_{KK}(s) - \frac{m_\pi^2}{6f^2}\bar{J}_{\eta\eta}(s) + \frac{4s}{f^2}\left\{L_5^r - \frac{1}{256\pi^2}\left(4\log\frac{m_\pi^2}{\mu^2} - \log\frac{m_K^2}{\mu^2} + 3\right)\right\} \ , \\
\widetilde{f}(s) &= \frac{3}{4}\,\frac{s}{f^2}\bar{J}_{KK}(s) + \frac{m_\pi^2}{3f^2}\bar{J}_{\eta\eta}(s) + \frac{12s}{f^2}\left\{L_4^r - \frac{1}{256\pi^2}\left(\log\frac{m_K^2}{\mu^2} + 1\right)\right\} \ ,
\end{aligned} \tag{17}
$$

and $\bar{J}_{PP}(s)$ $(P = \pi, K, \eta)$ is the standard meson loop function [9]

$$
\bar{J}_{PP}(s) = \frac{1}{16\pi^2}\left(2 + \sigma_P(s)\log\frac{\sigma_P(s) - 1}{\sigma_P(s) + 1}\right) \ , \tag{18}
$$

---

[2] We remark that more commonly the definition of these form factors includes the pertinent quark masses, such that e.g. the non-strange scalar form factor of the pion is defined via $\langle 0|\hat{m}\,(\bar{u}u + \bar{d}d)|\pi\pi\rangle = M_\pi^2\,\Gamma_\pi(s)$. For our later discussion, the overall normalization does not play a role but should be kept in mind.

and μ is the scale of dimensional regularization. The quantities μ_P in Eq.(16) are given by

$$\mu_P = \frac{m_P^2}{32\pi^2 f^2} \log \frac{m_P^2}{\mu^2} \ . \tag{19}$$

The scalar kaon form factors at next-to-leading order in CHPT are not given explicitly in the literature. We fill here this gap by performing such a calculation. This implies calculating the diagrams shown in Fig.3, which comprise the lowest order CHPT result, Fig.3a, already derived in section III A, the tadpole contribution, Fig.3b, and the unitarity corrections, Fig.3c. The vertices for these diagrams come from the lowest order CHPT Lagrangian. We note that wave function renormalization diagrams are not depicted in this figure. Finally, in Fig.3d, the local contribution coming from the $\mathcal{O}(p^4)$ CHPT Lagrangian is depicted. These terms are parameterized in terms of the scale–dependent, renormalized low energy constants $L_i^r(\mu)$ (in our case $i = 4, 5, 6, 8$). Evaluating these diagrams leads to

$$\begin{aligned}
\Gamma_2^n(s) &= \frac{1}{\sqrt{2}}\Bigg\{1 + \frac{4L_5^r}{f^2}(s-4m_K^2) + \frac{8L_4^r}{f^2}(2s-6m_K^2-m_\pi^2) + L_8^r\frac{32m_K^2}{f^2} + \frac{16L_6^r}{f^2}(6m_K^2+m_\pi^2) + \frac{2}{3}\mu_\eta \\
&\quad + \frac{9s-8m_K^2}{36f^2}J_{\eta\eta}^r(s) + \frac{3s}{4f^2}\left[J_{\pi\pi}^r(s)+J_{KK}^r(s)\right]\Bigg\} \ , \\
\Gamma_2^s(s) &= 1 + \frac{4L_5^r}{f^2}(s-4m_K^2) + \frac{8L_4^r}{f^2}(s-4m_K^2-m_\pi^2) + L_8^r\,\frac{32m_K^2}{f^2} + \frac{16L_6^r}{f^2}(4m_K^2+m_\pi^2) + \frac{2}{3}\mu_\eta \\
&\quad + \frac{9s-8m_K^2}{18f^2}J_{\eta\eta}^r(s) + \frac{3s}{4f^2}J_{KK}^r(s) \ .
\end{aligned} \tag{20}$$

As a test of our calculations we have checked that the infinities, associated with the wave function renormalization contributions and the loops in Fig.3c and 3d are properly absorbed by the infinite parts of the pertinent low energy constants and thus the expressions given in Eqs.(20) are finite.

C. Unitarity requirements

We now discuss the constraints that unitarity imposes on the scalar form factors. Of course, at low energies, one can simply work with CHPT and treat unitarity in a perturbative fashion. Here, however, we are interested also at energies of the order of 1 GeV, which requires some resummation technique and also the channel coupling between the $\pi\pi$ and the $K\overline{K}$ systems has to be taken into account. This has been elaborated in big detail in Ref. [14] and we present here the formalism necessary to discuss the scalar form factors. For convenience, we employ the matrix notation already introduced in the previous subsection, i.e. pions are labeled by the index 1 and kaons by the index 2. ¿From Ref. [14], we have the following expression for the T-matrix for meson–meson scattering,

$$T(s) = \left[I + K(s)\cdot g(s)\right]^{-1}\cdot K(s) \ , \tag{21}$$

where s denotes the centre-of-mass energy squared and $K(s)$ can be obtained from the lowest order CHPT Lagrangian,

$$\begin{aligned}
K(s)_{11} &= \frac{s-m_\pi^2/2}{f_\pi^2} \ , \\
K(s)_{12} &= \frac{\sqrt{3}s}{4f_\pi^2} \ , \\
K(s)_{22} &= \frac{3s}{4f_\pi^2} \ .
\end{aligned} \tag{22}$$

We remark that because of time reversal, both $K(s)$ and $T(s)$ are symmetric functions, so that $K(s)_{21} = K(s)_{12}$ and similarly for $T(s)$. The matrix $g(s)$ is also diagonal and given by [32]

$$g(s)_i = \frac{1}{16\pi^2}\left\{\sigma_i(s)\log\frac{\sigma_i(s)\sqrt{1+\frac{m_i^2}{q_{\rm max}^2}}+1}{\sigma_i(s)\sqrt{1+\frac{m_i^2}{q_{\rm max}^2}}-1} - 2\log\left[\frac{q_{\rm max}}{m_i}\left(1+\sqrt{1+\frac{m_i^2}{q_{\rm max}^2}}\right)\right]\right\} \ , \quad i = 1, 2 \ , \tag{23}$$

where $\sigma_i(s)=\sqrt{1-4m_i^2/s}$, $f_\pi \simeq 93$ MeV is the weak pion decay constant, m_i are the masses of the pions ($m_1=138$ MeV) and kaons ($m_2=495.7$ MeV) and $q_{\rm max}=0.9$ GeV is a cut-off in three-momentum space. On the other hand, $g(s)_i$ can also be calculated in dimensional regularization, using the standard $\overline{MS}-1$ scheme employed in CHPT [9],

$$g(s)_i=\frac{1}{(4\pi)^2}\left(-1+\log\frac{m_i^2}{\mu^2}+\sigma_i(s)\log\frac{\sigma_i(s)+1}{\sigma_i(s)-1}\right)=-J^r_{ii}(s)\ , \tag{24}$$

where the last equality follows from the definition of the renormalized two–meson loop function [9]

$$\bar{J}_{ii}(s)-\frac{1}{16\pi^2}\left(1+\log\frac{m_i^2}{\mu^2}\right)=J^r_{ii}(s)\ . \tag{25}$$

Note that we have changed the subscript "PP" appearing in Eq.(18) into "ii" to conform with our matrix notation. By expanding Eq.(23) in terms of $m_i/q_{\rm max}$ one can easily see, as discussed in appendix 2 of Ref. [32], that the differences between $g(s)_i$ in Eq.(23) and Eq.(24) are of higher order in the chiral expansion, i.e. of order $\mathcal{O}(m_i^2/q_{\rm max}^2)$, for the following value of the scale μ,

$$\mu=\frac{2q_{\rm max}}{\sqrt{e}}\simeq 1.2\,q_{\rm max}\ . \tag{26}$$

For energies above the threshold of the state i, unitarity implies the following relation between form factors and the $I=0$ T-matrix:

$$\text{Im }\Gamma_i(s)=\sum_j\Gamma_j(s)\frac{p_j(s)}{8\pi\sqrt{s}}\theta(s-4m_j^2)(T_{ji}^{\text{S-wave}}(s))^* \tag{27}$$

with $p_i(s)=\sqrt{s/4-m_i^2}$ the modulus of the c.m. three-momentum of the state i, and the strong amplitudes are projected on the S-wave. In the former equation we have suppressed the superscript "n" or "s", appearing in Eq.(15), since the previous equation applies to any of them. Finally, in what follows, we will also remove the superscript "S-wave" with the understanding that any partial wave is projected onto the S-wave. Taking now the complex conjugate on the right-hand-side of Eq.(27) and using the fact that the T-matrix is symmetric, we can rewrite Eq.(27) in matrix notation as:

$$\text{Im }\Gamma(s)=T(s)\cdot\frac{Q(s)}{8\pi\sqrt{s}}\cdot\Gamma^*(s) \tag{28}$$

where

$$Q(s)=\begin{pmatrix} p_1(s)\theta(s-4m_1^2) & 0\\ 0 & p_2(s)\theta(s-4m_2^2)\end{pmatrix}\ ,\quad \Gamma(s)=\begin{pmatrix}\Gamma_1(s)\\ \Gamma_2(s)\end{pmatrix}\ . \tag{29}$$

Substituting in the previous equation Im $\Gamma(s)$ by $(\Gamma(s)-\Gamma(s)^*)/(2i)$ and $T(s)$ by its expression given in Eq.(21), one has:

$$\begin{aligned}\Gamma(s)&=\left\{I+[I+K(s)\cdot g(s)]^{-1}\cdot K(s)\cdot i\frac{Q(s)}{4\pi\sqrt{s}}\right\}\cdot\Gamma(s)^*\\ &=[I+K(s)\cdot g(s)]^{-1}\cdot\left\{I+K(s)\cdot g(s)+K(s)\cdot i\frac{Q(s)}{4\pi\sqrt{s}}\right\}\cdot\Gamma(s)^*\ .\end{aligned} \tag{30}$$

Taking into account that the $K(s)$-matrix, Eq. (22), is real and that

$$g(s)^*=g(s)+i\frac{Q(s)}{4\pi\sqrt{s}} \tag{31}$$

since

$$\text{Im }g(s)=-\frac{Q(s)}{8\pi\sqrt{s}} \tag{32}$$

we can write Eq.(30) as:

$$[I + K(s) \cdot g(s)] \cdot \Gamma(s) = [I + K(s) \cdot g(s)^*] \cdot \Gamma(s)^* \ . \tag{33}$$

This tells us that the quantity $[I + K(s) \cdot g(s)] \cdot \Gamma(s)$ has no cuts since the only one which appears in $g(s)$ and $\Gamma(s)$, the right or unitarity cut, is removed. Therefore, we can express $\Gamma(s)$ as:

$$\Gamma(s) = [I + K(s) \cdot g(s)]^{-1} \cdot R(s) \tag{34}$$

with $R(s)$ being a vector of functions free of any singularity. We remark that this procedure of taking into account the final state interactions is based on the work presented in Ref. [33]. In the following, we will fix $R(s)$ by requiring the matching of Eq.(34) to the next-to-leading order (one loop) CHPT $\pi\pi$ and $K\bar{K}$ scalar form factors. These are calculated in the next subsection.

It is worth to stress that Eq.(34), given in terms of a vector of functions $R(s)$ without any cut, can be applied to any K-matrix without unphysical cut contributions, as the one derived in ref. [14]. The use of the strong amplitudes calculated from this reference is appealing for several reasons: 1) Because of their simplicity, 2) they have been already successfully used to describe many two meson production processes, as discussed in the introduction, and 3) higher order corrections to the kernel used in ref. [14] are not necessary to match with the next-to-leading order CHPT scalar form factors. In fact, in ref. [24] one can find a detailed comparison between the approach of ref. [14] and the more general ones described in refs. [24] and [32]. The main conclusion is that, apart from the detail of including (or not) the $\eta\eta$ channel as already discussed, the unitarity corrections coming from the rescattering of the lowest order CHPT kernel completely dominate the strong S-wave $I = 0$ scattering amplitudes up to about 1.2 GeV. Thus, one would not expect relevant departures from the use of the strong amplitudes from ref. [14] or from refs. [32] or [24]. In fact, all these approaches give rise to very similar pole positions for the $f_0(980)$ and σ mesons. For higher energies new effects have to be taken into account as e.g. the contributions from a pre-existing octet of scalar resonances around 1.4 GeV [24][3] and the increasingly important role played by multiparticle states, basically the 4π intermediate state. In addition, one has to deal with more relevant interaction vertices between the various fields than those given in Eq.(1) as discussed in section II.

D. Matching with chiral perturbation theory

The general expression for the scalar form factors given in Eq.(34) can be further constrained by matching it to the one loop CHPT expression given in Eqs.(16, 20). This ensures that for energies where CHPT is applicable, these form factors fulfill all requirements given by chiral symmetry and the underlying power counting. This matching procedure essentially fixes the vector $R(s)$. We remark that since in our unitarization procedure we are not considering the $\eta\eta$ channel we thus can not reproduce the chiral logarithms associated with this channel. Therefore, we will only consider the contribution form this channel to the value of the form factors at $s = 0$ and we will not include any s dependence. This approximation should not induce any sizeable theoretical error because the influence of the $\eta\eta$ channel was found to be significant only above its threshold [24,34] as already discussed in the introduction (when comparing the results of Ref. [14] and Ref. [24]).

We only discuss in detail the matching for the form factor $\Gamma_2^s(s)$. The procedure for the other form factors is completely analogous and we thus only give the final results for them. From Eq.(34) one has:

$$\Gamma^s(s) = [I + K(s) \cdot g(s)]^{-1} \cdot R^s(s) = [I - K(s) \cdot g(s)] \cdot R^s(s) + \mathcal{O}(p^4) \tag{35}$$

where the superscript "s" in $R^s(s)$ indicates that we are considering the $\bar{s}s$ form factor. From the former equation and Eq.(14) one has that $R^s(s)_1 = \mathcal{O}(p^2)$ and that $R^s(s)_2 = 1 + \mathcal{O}(p^2)$. Hence, we can recast Eq. (35) as:

$$\Gamma^s(s)_2 = R^s(s)_2 - K(s)_{22}\, g(s)_2 + \mathcal{O}(p^4) = R^s(s)_2 + \frac{3s}{4f^2}\, J^r_{KK}(s) + \mathcal{O}(p^4) \tag{36}$$

at the regularization scale $\mu = 1.2 q_{\max}$. Comparing this result with the one given in Eq.(20) leads to

$$\begin{aligned} R^s(s)_2 = 1 &+ \frac{4\,L_5^r}{f^2}(s - 4m_K^2) + \frac{8L_4^r}{f^2}(s - 4m_K^2 - m_\pi^2) + L_8^r \frac{32 m_K^2}{f^2} + \frac{16 L_6^r}{f^2}(4m_K^2 + m_\pi^2) + \frac{2}{3}\mu_\eta \\ &+ \frac{m_K^2}{36\,\pi^2\, f^2}(1 + \log \frac{m_\eta^2}{\mu^2}) \ , \end{aligned} \tag{37}$$

[3] Preexisting means here that these resonances with a mass around 1.4 GeV are as "elementary" as the basic fields π, K or η.

using the Gell-Mann–Okubo relation $m_\eta^2 = 4m_K^2/3 - m_\pi^2/3$, the deviations from it being of higher order for our purpose. Proceeding in an analogous way for the other form factors one concludes:

$$\begin{aligned}
R^n(s)_1 &= \sqrt{\frac{3}{2}}\Bigg\{1 + \frac{4(L_5^r + 2L_4^r)}{f^2}s + \frac{16(2L_8^r - L_5)}{f^2}m_\pi^2 + \frac{8(2L_6^r - L_4^r)}{f^2}(2m_K^2 + 3m_\pi^2) \\
&\quad - \frac{m_\pi^2}{32\,\pi^2 f^2} - \frac{1}{3}\mu_\eta\Bigg\}\;, \\
R^n(s)_2 &= \frac{1}{\sqrt{2}}\Bigg\{1 + \frac{4L_5^r}{f^2}(s - 4m_K^2) + \frac{8L_4^r}{f^2}(2s - 6m_K^2 - m_\pi^2) + L_8^r\,\frac{32m_K^2}{f^2} + \frac{16L_6^r}{f^2}(6m_K^2 + m_\pi^2) + \frac{2}{3}\mu_\eta \\
&\quad + \frac{m_K^2}{72\,\pi^2 f^2}(1 + \log\frac{m_\eta^2}{\mu^2})\Bigg\}\;, \\
R^s(s)_1 &= \sqrt{3}\left\{\frac{4L_4^r}{f^2}(s - 2m_\pi^2) + \frac{16L_6^r}{f^2}m_\pi^2\right\}\;. \qquad (38)
\end{aligned}$$

Notice that $R^s(s)_1$ is subleading in large N_c, i.e. of $\mathcal{O}(N_c^{-1})$, while the other quantities in Eqs.(37,38) are of order $\mathcal{O}(1)$ in this counting. This is expected since the production of pions from an $\bar{s}s$ source is subleading in large N_c QCD [5]. We also see, as already stressed in section III A that $R^s(s)_1$ is $\mathcal{O}(p^2)$ in the chiral counting. Once the functions $R^{n,s}(s)$ have been determined, the final expressions for the form factors are obtained by making use of Eq.(34, 22, 23). Finally, one has to take into account that, when using Eqs.(37, 38), the regularization scale is $\mu = 1.2q_{\rm max} \simeq 1.08$ GeV. Therefore, we have to run the low energy constants $L_i^r(\mu)$ to this scale from the usual ones $\mu = m_\eta$ or $\mu = m_\rho$, with m_η, m_ρ the mass of the η, ρ meson, respectively, by using the appropriate β–functions given in Ref. [9].

IV. RESULTS

We will first discuss the results for the $J/\Psi \to \phi\pi\pi(K\bar{K})$ decays and then we will also consider to some extent the $J/\Psi \to \omega\pi\pi$ decay. To be more specific, we consider the S–wave contribution to these decay modes.

A. The ϕ-meson case

Considering the phase space of three particles [25] we can write the unpolarized event distribution for the $J/\Psi \to \phi\pi^+\pi^-(K^+K^-)$ reactions as:

$$\begin{aligned}
\frac{d\,N(W)_i}{d\,W} &= \frac{\mathcal{C}_\phi^2\,G_i^2}{(2\pi)^3 12\,m_{J/\Psi}^2}\,|\Gamma_i^s(s) + \lambda_\phi\,\Gamma_i^n(s)|^2\left[1 + \frac{(m_{J/\Psi}^2 + m_\phi^2 - W^2)^2}{8m_\phi^2 m_{J/\Psi}^2}\right] \\
&\quad \times \sqrt{[W^2 - 4m_i^2]\left[(m_{J/\Psi}^2 - W^2 - m_\phi)^2 - 4m_\phi^2 W^2\right]}, \qquad (39)
\end{aligned}$$

where $i = 1$ refers to the $\pi^+\pi^-$ and $i = 2$ to the K^+K^- system, in order. Furthermore, W is the total energy in the c.m. of the two pions or kaons, G_i is basically a Clebsch-Gordan coefficient equal to 4/3 for pions and 1/2 for kaons, respectively, and $\mathcal{C}_\phi$ a normalization constant depending on the experiment, in our case DM2 [31] or MARK-III [35]. In comparing with the experimental data, we will average Eq.(39) over the width of the bin (as given by the corresponding experiment). As discussed in section III D, our calculated form factors depend on the CHPT low energy constants L_4^r, L_5^r, L_6^r and L_8^r. From these, only L_5^r and L_8^r are relatively well determined. Their most recent values, given in Ref. [12] from an $\mathcal{O}(p^6)$ CHPT analysis of the $K_{\ell 4}$ form factors, are:

$$10^3\,L_5^r(M_\rho) = 0.65 \pm 0.12\;, \quad 10^3 L_8^r(M_\rho) = 0.48 \pm 0.18. \qquad (40)$$

At the scale $\mu = 1.2q_{\rm max} \simeq 1.08$ GeV they are:

$$10^3\,L_5^r(1.08\text{ GeV}) = -0.15 \pm 0.12\;, \quad 10^3\,L_8^r(1.08\text{ GeV}) = 0.26 \pm 0.18\;, \qquad (41)$$

On the other hand, L_4^r and L_6^r are only poorly known and their present values [9] can be considered as stemming more from an estimation of their order of magnitude than from a truly phenomenological fit. According to Ref. [9], it is estimated that for a regularization scale μ between 0.5 and 1.0 GeV one should have $10^3\, L_4^r \simeq \pm 0.5$ and $10^3\, L_6 \simeq \pm 0.3$. A more recent determination [36] gives $10^3\, L_4^r(m_\rho) = -0.3 \pm 0.5$ and $10^3\, L_6^r(m_\rho) = -0.2 \pm 0.3$ so that at a scale of 1.08 GeV one has

$$10^3\, L_4^r(1.08 \text{ GeV}) = -0.57 \pm 0.5 \ , \quad 10^3\, L_6^r(1.08 \text{ GeV}) = -0.36 \pm 0.3 \ . \tag{42}$$

Again, this estimate relies on OZI (large N_c) arguments. To be more precise, one sets $L_{4,6}^r$ to zero at the scale $\mu = m_\eta$, which is of course somewhat arbitrary. One can also make use of information about the low energy coupling constant ℓ_4^r coming from two flavor CHPT by means of the relation [9], $\ell_4^r(\mu) = 8L_4^r(\mu) + 4L_5^r(\mu) - \nu_K/2 + \mathcal{O}(p^6)$ with $\nu_K = [\ln(M_K^2/\mu^2) + 1]/32\pi^2$. The low energy coupling constant ℓ_4^r has been determined at $\mathcal{O}(p^4)$ in ref. [9] with the result $10^3\, \ell_4^r = 1.2 \pm 6$ at the scale $\mu = 1.08$ GeV. However, in ref. [13] making use of the analytically deduced pion scalar form factor $\bar{u}u + \bar{d}d$ up to and including $\mathcal{O}(p^6)$, they update the previous value and give the improved result $10^3\, \ell_4^r = 1.8 \pm 1.9$ at the same scale. Although the central value from both determinations is very similar the error is much smaller in the second case. With the value for L_5^r given in Eq. (41), we obtain $10^3\, L_4^r(1.08 \text{ GeV}) = 0.1 \pm 0.7$ when using ℓ_4^r from ref. [9] and

$$10^3\, L_4^r(1.08 \text{ GeV}) = 0.19 \pm 0.25 \tag{43}$$

in the case of $\mathcal{O}(p^6)$ SU(2) CHPT [13]. The value obtained in this way for L_4^r is also consistent with zero, but on the positive side, in stark contrast to the value given in Eq.(42) whose central value is very far from the more precise determination coming from refs. [12,13] as given in Eq.(43). Thus, although at present no precise determination of this low energy constant is available, rather strong constraints on its value can be obtained by combining the determination of the low energy constants making use of $\mathcal{O}(p^6)$ CHPT, both in its SU(2) [13] and SU(3) [12] forms.

More recently, some constraints on the couplings L_4^r and L_6^r have been reported [10,11,7]. The determination of L_4^r relies on a comparison of the CHPT series at next- [10] and next-to-next-leading order [11] with a phenomenologically determined scalar form factor. In ref. [10] the value $10^3\, L_4^r(1.08 \text{ GeV}) \simeq 0.14$ is given without errors and the band of values $-0.12 \leq 10^3\, L_4^r(1.08 \text{ GeV}) < 0.04$ is reported in ref. [11]. We note that the second set of values for L_4^r [11] is in the lower limit of the value for L_4^r given in Eq.(43). Nevertheless, the determination of L_6^r [10,11] is not so well sounded due to strong simplifying working assumptions when computing phenomenologically the quark correlator $(\bar{u}u + \bar{d}d)\bar{s}s$ [10,11]. It is also stated in ref. [10] that the positivity of the fermionic measure gives rise to a lower bound for L_6^r, $10^3 L_6^r(1.08 \text{ GeV}) \geq 0.03$. However, this bound is somewhat arbitrary since the only necessary requirement to make use of the positivity of the fermionic measure is that all the three light quark masses have to be equal. In ref. [10], they were set equal to the strange quark mass, but they could as well have been taken on another value. In fact, for the average light quark masses $(m_u + m_d)/2$, the corresponding lower bound is: $10^3 L_6^r(1.08 \text{ GeV}) \geq -0.75$. The bound based on using the strange quark mass can only be maintained if one assumes the next–to–leading order corrections to be of canonical size, $cM_K^2/(4\pi f_\pi)^2$, with c a number of order one.[4]

While the data of DM2 [31] have been published, this is not the case for the data of MARK-III [35]. On the other hand, both experiments, see Figs.7 and 8, are compatible for the $\pi^+\pi^-$ event distribution for 25 and 10 MeV bins[5]. However, this is not the case for the $K\overline{K}$ event distribution, see Fig.9. Consequently, we will only consider for our fits the data from DM2 [31] for the $\pi^+\pi^-$ event distributions in the $J/\Psi \to \phi\pi^+\pi^-$ decay, both for the 10 and the 25 MeV bins. In fitting the data for the $J/\Psi \to \phi\pi^+\pi^-$ distribution from DM2 we will fix L_5^r and L_8^r at the values given in Eq.(41). On the other hand, L_4^r and L_6^r will be taken as free parameters. In this way, our expression for the event distribution of the pions and kaons will have four free parameters: $\mathcal{C}_\phi$, λ_ϕ, L_4^r and L_6^r. However, there is still too much freedom in fitting the data with this set of free parameters since the global normalization constant $\mathcal{C}_\phi$ can only be determined within a large range and with a sizeable uncertainty. We will further restrict our fit by requiring that we can also describe the pion event distribution in the $J/\Psi \to \omega\pi^+\pi^-$ decay, at least in the low energy region where the S-wave contribution, the one which we are considering here, is dominant. This extension of our model to the ω case is discussed in the next subsection. Imposing this requirement, $\mathcal{C}_\phi$ is fixed,[6] $\mathcal{C}_\phi = (16 \pm 3)\,\text{MeV}^{-1}$, and the fitted values for λ_ϕ, $L_4^r(1.08 \text{ GeV})$ and $L_6^r(1.08 \text{ GeV})$ are:

[4]We are grateful to Bachir Moussallam for a claryfing discussion on this topic.

[5]The data for the 10 MeV bins of MARK-III has been taken from Ref. [1].

[6]There is approximately a factor 1.11 between the global normalization constant required for the DM2 data with respect the one required for MARK-III.

$$\lambda_\phi = 0.17 \pm 0.06 \ , \quad 10^3 \, L_4^r(1.08 \text{ GeV}) = 0.44 \pm 0.11 \ , \quad 10^3 \, L_6^r(1.08 \text{ GeV}) = -0.38 \pm 0.06 \ , \tag{44}$$

with a $\chi^2/\text{dof} = 0.92$. Clearly $\lambda_\phi \neq 0$, in contradiction with the OZI rule. Furthermore, the pion event distribution turns out to be very sensitive to the large N_c subleading low energy constants L_4^r and L_6^r. The theoretical uncertainties given in Eq.(44) are obtained in the following way. We have allowed for a relative change of 20% in the global normalization constant $\mathcal{C}_\phi$ when considering the data with the ϕ and also the ones with the omega in the final state. We consider this estimate of the error in $\mathcal{C}_\phi$ as conservative, since the ensuing deviation from the $\omega\pi^+\pi^-$ data for such changes in $\mathcal{C}_\phi$ is larger than the uncertainty in the omega data by assuming a Poisson distribution. On the other hand, we have also allowed an uncertainty of ± 0.1 GeV in the determination of $q_{\rm max}$ from ref. [14] and then we calculated the band of values for λ_ϕ, L_4^r and L_6^r. All these sources of uncertainty are added in quadrature together with the statistical error given by the fitting procedure when using the central values for $\mathcal{C}_\phi$ and $q_{\rm max}$.

It is instructive to compare the values for the low energy constants found here, e.g. Eqs.(44), with the ones given in Eq.(42). While L_6^r agrees perfectly within error bars, the sign of L_4^r is changed. Stated differently, if we evaluate from Eq. (44) $10^3 \, L_4^r$ at the rho mass, we find a value of 0.71, which is sizeably larger in magnitude than the central value given in Ref. [36]. Also, it is larger than the positive value deduced from SU(2) information given in Eq.(43), at the scale $\mu = m_\rho$ one has for this case $10^3 \, L_4^r = 0.46 \pm 0.25$, although both values are consistent within errors. We reiterate that using large N_c arguments, one would expect L_4^r to be zero (at a scale somewhere in the resonance region). Therefore, our increased value and also the one from Eq.(43), clearly signals OZI violation. Quite differently, our value for $10^3 \, L_6^r(m_\rho) = -0.22$ is completely consistent with the previous determination [36]. However, it does not fulfill the positivity constraint $10^3 \, L_6^r(m_\rho) \geq 0.20$ [7,10] but it fulfills the other reasonable 'positivity' bound previously discussed $10^3 \, L_6^r(m_\rho) \geq -0.59$. In fact, our value for $10^3 \, L_6$ lies in an natural intermediate region between both extreme lower bound. Nevertheless, we have also performed a series of fits enforcing the former constraint. We can fit the ϕ data, but on the expense of very large values for λ_ϕ and L_4^r. Furthermore, it is not possible to simultaneously get a description of the ω decay data. We think that further study is needed in order to apply the positivity of the Dirac measure and also, we should stress that the LEC L_6^r is plagued by the Kaplan–Manohar ambiguity [37]. It is important to point out that one can criticize our determinations of the low energy constants for two reasons. First, our model for the J/Ψ decay with the ϕ meson as a spectator is fairly simple, one could e.g. write down higher order transition operators which would complicate the analysis. Given, however, the fact that we can precisely reproduce the data both for the ϕ and the ω resonances, it is not obvious a priori that such a modified ansatz would lead to very different results. Second, the use of unitarity to determine the scalar form factors beyond one loop accuracy induces some inevitable model dependence. To overcome this, one could think of doing a pure CHPT analysis on the left wing of the scalar resonance. We believe, however, that the present data in this energy region are not precise enough for an accurate determination of the LECs. Independently of these reservations, our analysis clearly underlines that the OZI rule is strongly violated in the scalar 0^{++} sector, as indicated e.g. by the large positive value of the LEC $L_4^r(m_\rho)$ and also by the non-vanishing value of λ_ϕ. With respect the latter point, see also the footnote in section IV B.

It is also worth to indicate that in ref. [34], making use of the Inverse Amplitude Method (IAM) [40] with complete next-to-leading order CHPT strong amplitudes, a fit to the $I = 0$ and 2 S-wave and $I = 1$ P-wave $\pi\pi$ and $K\overline{K}$ partial wave amplitudes was done in terms of the low energy coupling constants L_1^r, L_2^r, L_3^r, L_4^r, L_5^4 and $2L_6^r + L_8^r$. This study has in common with the present one that a complete matching to the relevant next-to-leading order CHPT results was given and at the same time fully unitarity amplitudes were derived. The experimental data was very well reproduced up to energies around 1.2 GeV giving rise to the presence of the resonances ρ and $f_0(980)$. The values obtained for L_5^r and $2L_6^r + L_8^r$, within errors, are consistent with those recently obtained in ref. [12]. This implies agreement of the results of that reference with our choice for the values of L_5^r and L_8^r [12] and our presently determined value for L_6^r. The main difference between the set of values given in ref. [34] and those in ref. [12] corresponds to the value of L_2^r. While in the former case $L_2^r \simeq 2\, L_1^r$ as required by Vector Meson Dominance (VMD) [7] [42], this relation is only fulfilled within errors by the values given in ref. [12]. Nevertheless, in ref. [34] $10^3 \, L_4^r(m_\rho) = 0.2 \pm 0.1$. This value, although on the positive side, is incompatible with our present one, $10^3 \, L_4^r(m_\rho) = 0.71 \pm 0.11$, and, within errors, is compatible with the rest of analyses presented in this section except for [11]. Summarizing, for L_6^r there is a rather good agreement between our present study and refs. [16], [12], [34] in disagreement with the finding of ref. [10,11]. On the other hand, for L_4^r our present analysis is compatible only with that value of L_4^r determined from $\mathcal{O}(p^6)$ CHPT [13,12] which also find quite a sizeable central value at $\mu = M_\rho$, around 0.5×10^{-3}, different from the smaller numbers of refs. [16,10,11,34].

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[7] There is a very close link between VMD and the IAM in the vector channels [41,24].

The resulting non–strange and strange normalized scalar form factors of the pion and the kaon are shown in Fig.4, Fig.5 and Fig.6. In the case of the non–strange scalar form factor, we show for comparison in Fig.4 the one– and two–loop CHPT [16,38] as well as the dispersion theoretical results [39] and the exponentiated two–loop CHPT result [38]. In the latter case, the two–loop CHPT result is improved by making use of an Omnès resummation in terms of the next-to-leading order CHPT phase shifts. Our result is close to the ones obtained by a different method in Ref. [39] and even closer to the exponentiated two–loop CHPT results of ref. [38]. The agreement is worse when comparing our results with the so called modified-Omnès representation of refs. [38,13]. We also remark that the two–loop representation covers the main feature of this quantity below $W \simeq 600$ MeV, as it is known since long [38]. The strange scalar form factor of the pion is reasonably well described for energies below 350 MeV. In contrast, the strange and the non–strange scalar form factor of the kaon are poorly described at one loop, as expected from the larger mass of the kaon.

In Figs.7 and 8, we show the curves from ours fit to the $\pi^+\pi^-$ event distribution in comparison with the experimental data from DM2 and MARK-III for the 25 and the 10 MeV bins, respectively. The data of MARK-III have been multiplied by the factor $\mathcal{C}^2_{\phi,\mathrm{DM2}}/\mathcal{C}^2_{\phi,\mathrm{MARK-III}} \simeq 1.11^2$ in order to facilitate the comparison between both sets of data. The agreement with the experimental data is very good as indicated by the low χ^2/dof of 0.92. By comparing Fig.7 with the left panel of Fig.6 we see that the event distribution of the two pions is dominated by the strange scalar form factor of the pion. Moreover, in Fig.7 the changes in the results when allowing a change in the cut-off q_{max} by ± 0.1 GeV [14] are shown to be quite small. In Fig.9 our prediction for the K^+K^- event distribution is depicted. Incidentally, we find better agreement with the data of MARK-III than with the ones of DM2 with respect to this decay mode. This was also noted in Refs. [1,3]. In fact, in these references a fit of similar quality to the data of DM2 and MARK-III is also given. The important difference between their method and ours is that we have devised a dynamical approach which means that the parameters that enters in our description of the problem are not specific to it and can be related to many other physical observables. This is particularly true for L_4^r and L_6^r. But even for λ_ϕ we will see in the next subsection how it can be related to the whole set of U(3) processes that follows from the decays of the J/Ψ resonance to any vector resonance belonging to the lightest nonet of vector resonances and two pseudoscalars. On the other hand, in ref. [1,3] no attempt was done to describe the $J/\Psi \to \omega\pi\pi$ decays.

B. The ω-meson case

A priori one can expect that the $J/\Psi \to \omega\pi\pi$ decay requires a rather different dynamical description than that for the mode $J/\Psi \to \phi\pi\pi$ considered so far. For instance, the approach of considering the ω as a spectator is by no means so clear as for the ϕ case. Note that the Dalitz plot for this decay has very clear bands due to the decays $b_1(1235) \to \omega\pi$ and $f_2(1270) \to \pi^+\pi^-$ [35]. The latter induces a sizeable D-wave contribution so that our approach can only be applied for the first few hundred MeV of the two pion event distribution. However, making use of SU(3) symmetry, we can extend our considerations from section II and we will present our calculated S-wave contribution to the invariant mass distribution of the pions in the $J/\Psi \to \omega\pi\pi$ decay. This calculation is completely fixed in terms of the parameters already given for the ϕ case, cf. Eq.(44), except for the global normalization constant for which we have also used the experimental data from the $J/\Psi \to \omega\pi^+\pi^-$ decays to further constraint its value.

Let us now be more specific and work out the aforementioned relation between the two cases. An invariant SU(3) Lagrangian involving the octet and singlet of vector resonances $V_\mu^{(8)}$ and $V_\mu^{(1)}$, respectively, and the corresponding ones of the scalar sources $S^{(8)}$ and $S^{(1)}$ can be written as (making also use of Lorentz invariance)

$$\mathcal{L} = \hat{g}\left(\Psi^\sigma \langle V_\sigma^{(8)} S^{(8)}\rangle + \nu \Psi^\sigma V_\sigma^{(1)} S^{(1)}\right) , \tag{45}$$

where $\hat{g}$ is an overall coupling constant whose precise value is not needed in the following. We only need to determine the relative strength of the octet to singlet couplings given in terms of the real parameter ν. The symbol $\langle ... \rangle$ refers to the trace over the SU(3) indices of the matrices $V^{(8)}$ and $S^{(8)}$. These are defined via

$$V_\sigma^{(8)} = \begin{bmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}V_8 & \rho^+ & K^{*\,+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}V_8 & K^{*\,0} \\ K^{*\,-} & \bar{K}^{*\,0} & -\frac{2}{\sqrt{6}}V_8 \end{bmatrix}_\sigma . \tag{46}$$

and similarly for the $S^{(8)}$ matrix. In the previous equation we have denoted by V_8 the $I = 0$ state of the octet of vector resonances. This formalism is in close analogy with the one used in CHPT for the octet of pseudoscalars,

compare Eq.(11). Denoting by S_8 the $I = 0$ operator of the octet of scalar sources, we can write the terms involving V_8 and V_1 of Eq.(45) as:

$$\Psi^\sigma \left(V_{8\,;\sigma} S_8 + \nu V_\sigma^{(1)} S^{(1)} \right) \ . \tag{47}$$

Considering ideal mixing[8] between the V_8 and the $V^{(1)}$ then:

$$V_8 = \frac{\omega}{\sqrt{3}} - \sqrt{\frac{2}{3}}\phi \ , \quad V^{(1)} = \sqrt{\frac{2}{3}}\omega + \frac{\phi}{\sqrt{3}} \ . \tag{48}$$

In an analogous way we will also introduce the scalar sources S_ω and S_ϕ defined by

$$S_8 = \frac{S_\omega}{\sqrt{3}} - \sqrt{\frac{2}{3}}S_\phi \ , \quad S^{(1)} = \sqrt{\frac{2}{3}}S_\omega + \frac{S_\phi}{\sqrt{3}} \ . \tag{49}$$

Note that in a quark model language, consistently with the transformation properties under SU(3), we can write:

$$S_\phi = \bar{s}s \quad \text{and} \quad S_\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \ . \tag{50}$$

Rewriting Eq.(47) in terms of ω, ϕ, S_ω and S_ϕ, we have:

$$\frac{2+\nu}{3}\Psi^\sigma \phi_\sigma \left(S_\phi + S_\omega \frac{\sqrt{2}(\nu-1)}{2+\nu} \right) + \frac{\sqrt{2}(\nu-1)}{3}\Psi^\sigma \omega_\sigma \left(S_\phi + S_\omega \frac{1+2\nu}{\sqrt{2}(\nu-1)} \right) \ . \tag{51}$$

In this way, the parameter λ_ϕ introduced in Eqs.(3,4) and fitted in the previous subsection, can now be expressed as:

$$\lambda_\phi = \frac{\sqrt{2}(\nu-1)}{2+\nu} \ . \tag{52}$$

¿From this equation we can isolate $\nu = \nu(\lambda_\phi)$ and then predict the corresponding parameter λ_ω,

$$\lambda_\omega = \frac{1+2\nu(\lambda_\phi)}{\sqrt{2}(\nu(\lambda_\phi)-1)} \tag{53}$$

We can also obtain from Eq.(51) the global normalization constant for the ω in terms of the one of the ϕ since

$$\frac{C_\omega}{C_\phi} = \frac{\sqrt{2}(\nu-1)}{2+\nu} = \lambda_\phi \ . \tag{54}$$

In Fig.10 we show the calculated S-wave contribution to the event distribution of pions in the $\Psi \to \omega\pi^+\pi^-$ decay. This calculation does not introduce any new free parameter since all of them have been fixed in terms of the one of the ϕ case by Eqs.(53) and (54). The description of the data of MARK-III up to around 0.7 GeV is very good. For higher energies the D-wave contribution cannot be further neglected.

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[8] In Ref. [45] the departure from the ideal mixing in the $\omega - \phi$ system is thoroughly studied comparing different models, and is described by a rotation of the ideal mixing states with a rotation angle $|\delta_V| \approx 3^0$. This departure would produce corrections of the order of a 5% with respect the ideal mixing situation considered here. However, this value should be compared with the departure from 1 of the parameter ν which from Eq. (52), taking into account the value of λ_ϕ given in Eq. (44), is about 40%, a much bigger effect than any expected deviation from the ideal mixing situation in the $\omega - \phi$ system. Note that $\nu = 1$ is the value expected from U(3) symmetry and for this case $\lambda_\phi = 0$ and $\lambda_\omega = \infty$, see Eqs. (52,53)

V. CONCLUSIONS

In this work we have addressed the problem of the J/Ψ decays into a vector (ϕ, ω) and two pseudoscalar mesons (Goldstone bosons) measured at DM2 [31] and MARK-III [35]. These processes are considered to be mediated by the corresponding scalar form factors of the pseudoscalar mesons if one considers the emitted vector meson as a spectator. Consequently, these reactions are rather interesting since they are very sensitive to OZI violation physics, in our scheme parameterized by the constants λ_ϕ, see Eq.(4), and the low energy constants L_4^r and L_6^r of chiral perturbation theory. The first of these constants parameterizes the direct admixture of non–strange quarks to the scalar interpolating field for our model of the J/Ψ decay with the ϕ playing the role of a spectator, see Figs.1,2. The two low energy constants enter the one loop description of the pion and kaon scalar form factors. To describe these properly for the range of energies relevant here, we have combined information coming from next-to-leading order (one loop) chiral perturbation theory (CHPT) with the unitarity requirements which are valid to all orders in the chiral expansion. In addition, we also have calculated for the first time the next-to-leading order CHPT kaon scalar form factors, for strange, $\bar{s}s$, and non–strange, $\bar{u}u + \bar{d}d$, scalar–isoscalar quark densities. The unitarity requirements were imposed by using the strong $I = 0$ $\pi\pi$ and $K\overline{K}$ amplitudes derived in Ref. [14]. The amplitudes given in that paper not only describe accurately the S-wave $I = 0$ and $I = 1$ strong scattering data but also have been used to successfully reproduce or even predict experimental data for the whole set of reactions listed in the Introduction. With this input, we have successfully described, from threshold up to around 1.2 GeV,[9] the event distribution of the $\pi^+\pi^-$ system in the $J/\Psi \to \phi\pi^+\pi^-$ decay. We have then predicted, in agreement with the data from MARK-III, the event distribution of kaons in the $J/\Psi \to \phi K^+K^-$ reaction and the low energy part, where the S-wave dominates, of the event distribution of $\pi^+\pi^-$ pairs in the $J/\Psi \to \omega\pi^+\pi^-$ decay. Furthermore, the OZI violation parameter λ_ϕ comes out different from zero. This also holds for the low energy constants L_4^r and L_6^r. While the value of the latter agrees with previous estimates [16,34], our result for L_4^r is sizeably larger in magnitude as most previous estimations [16,10,11,34]. However, it is compatible within errors with the quite constraint value derived by combining the information from $\mathcal{O}(p^6)$ SU(2) [13] and SU(3) [12] CHPT. This offers another indication that the OZI rule does not account for the physics in the scalar 0^{++} channel, as stressed e.g. in Refs. [6–8]. The scheme employed here offers a unique approach to describe the scalar sector, which has been at the heart of many investigations over the last decade.

Clearly, to further improve the approach presented here, it would be mandatory to not only have event distributions but rather normalized data. This would allow one to pin down the low energy constants L_4^r and L_6^r more precisely together with the OZI violation parameter λ_ϕ as well as the product $(\hat{g}\, m_q\, B_0)^2$ (i.e. normalization of the scalar form factors times the strength of the scalar source to vector meson coupling). With respect to the former, one could also try to do a pure one (or even two) loop CHPT calculation for small two–pion invariant masses, i.e. on the left wing of the $f_0(980)$ resonance. Clearly, the presently available data are not precise enough for successfully doing that, but such a program is in principle of the similar interest as the study of chiral dynamics in τ decays, see e.g. Refs. [43,44], specially when referring to the scalar sector.

Acknowledgments

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[9] In principle, one could also go to higher energies but that is much more difficult due a) to the appearance of multiparticle states and b) due to novel interaction vertices as discussed in section II.

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FIGURES

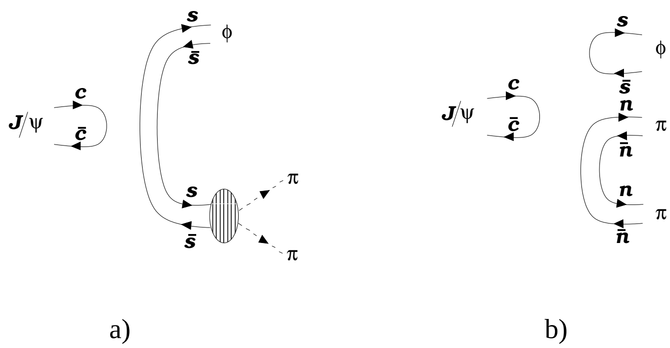

FIG. 1. Quark line diagrams for decay J/Ψ into the ϕ and a meson pair ($\pi\pi$ or $K\overline{K}$). The quark flavors are explicitly given, n refers to the light non–strange u, d quarks. The hatched blob in a) depicts the final state interactions in the coupled $\pi\pi/K\overline{K}$ system.

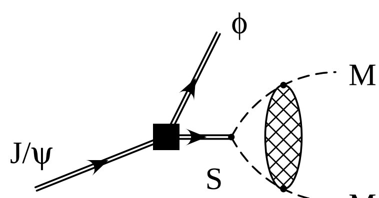

FIG. 2. Anatomy of the J/Ψ decay into a ϕ and a Goldstone boson pair. S is the interpolating scalar field described in the text. The cross–shaded blob symbolizes the final state interactions in the coupled $\pi\pi/K\overline{K}$ system.

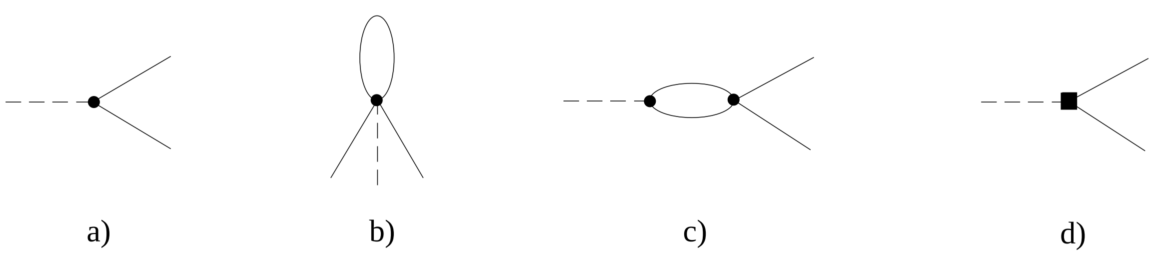

FIG. 3. Feynman diagrams for the calculation of the scalar form factors at leading and next-to-leading order in CHPT. The scalar source is indicated by the dashed line and the solid lines refer to the psuedoscalars (pions and kaons). From left to right: a) Lowest order, b) tadpole contributions, c) unitarity contributions and d) local contact terms from the $\mathcal{O}(p^4)$ CHPT Lagrangian. The full circles indicates that the vertices come from the lowest order CHPT Lagrangian and the full squares symbolize an insertion from next-to-leading order. The wave function renormalization diagrams are not drawn.

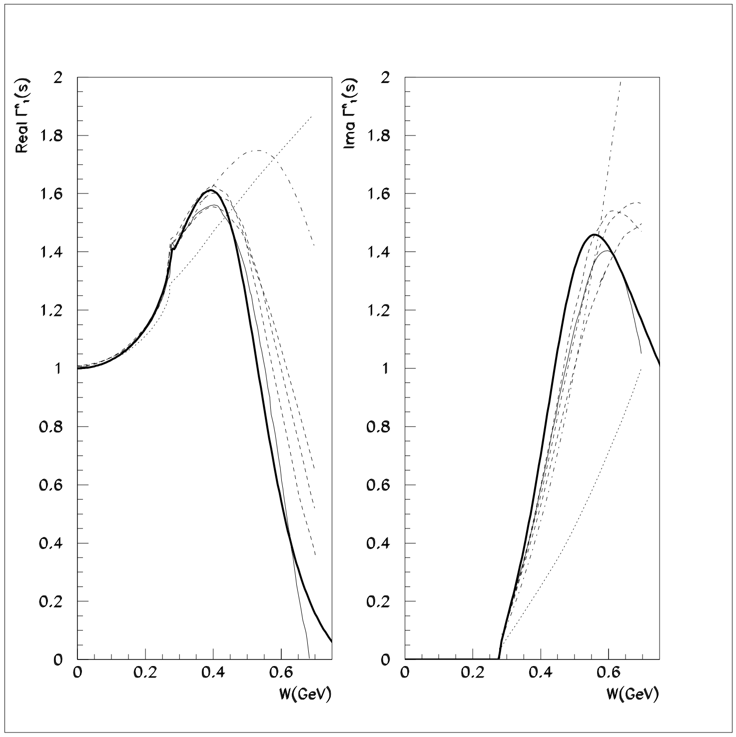

FIG. 4. Normalized non–strange scalar form factor of the pion (wide solid line). Dotted and dot–dashed lines: One and two loop CHPT results, respectively. The three dashed lines are the dispersion theoretical results from Ref. [39]. The thin solid lines represent the exponentiated two–loop CHPT results [38]. Our results correspond to the thick solid lines. Left/right panel: Real/imaginary part.

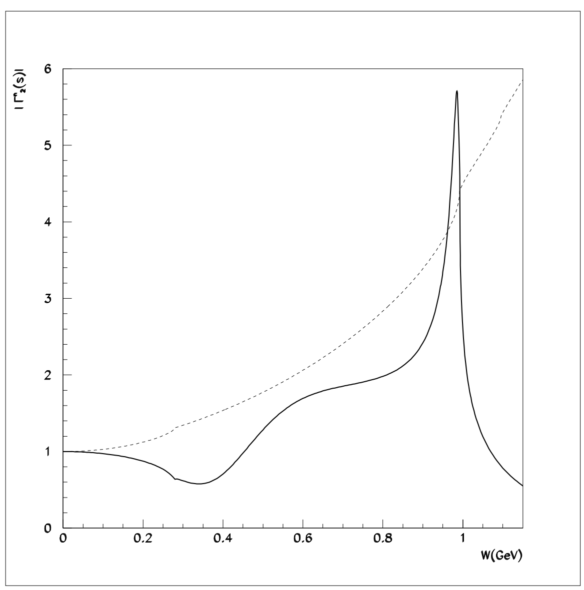

FIG. 5. Normalized non–strange scalar form factor of the kaon (solid line) in comparison to the one loop CHPT result (dashed line).

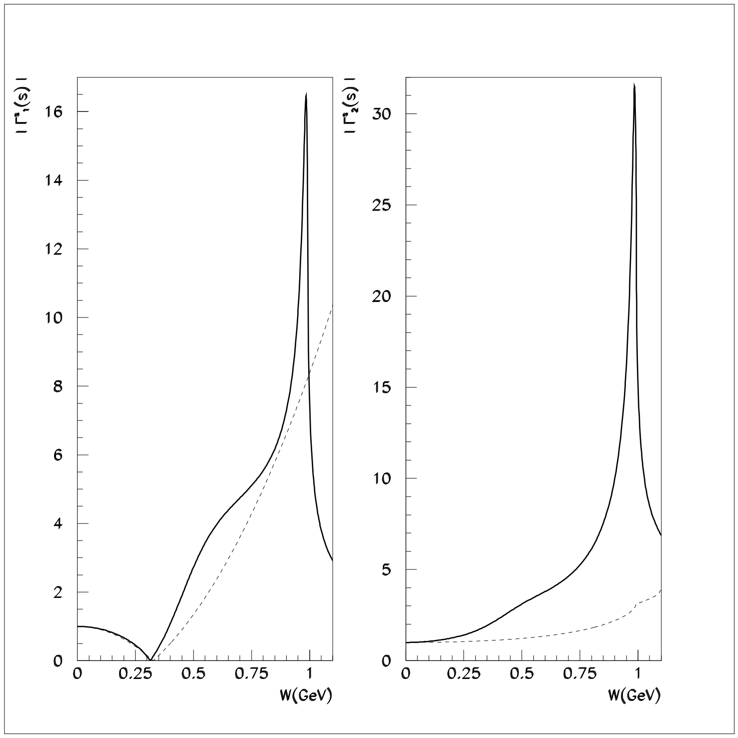

FIG. 6. Normalized strange scalar form factor of the pion (left panel) and of the kaon (right panel). Solid lines: Chiral unitary approach. Dashed lines: one loop CHPT result.

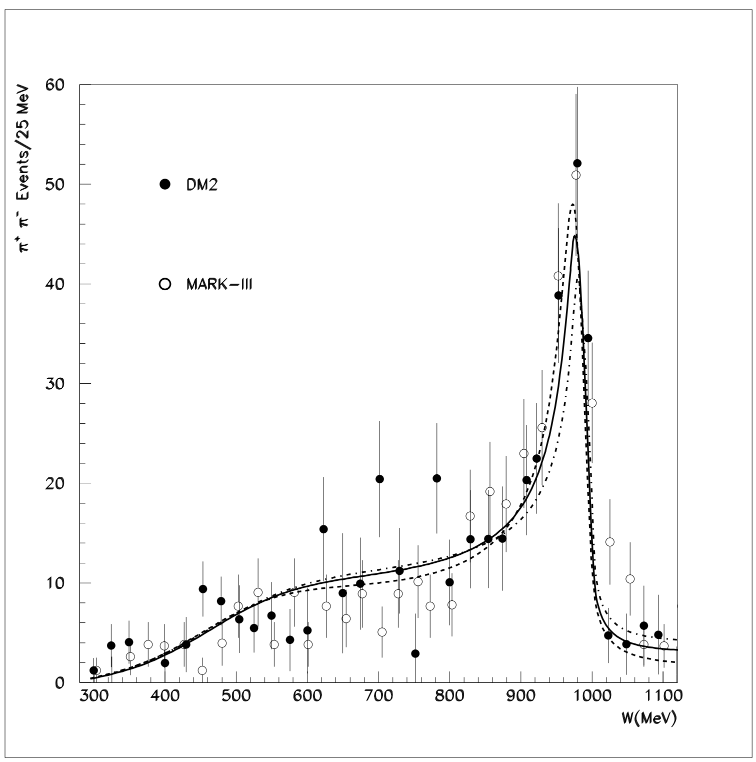

FIG. 7. $\pi^+\pi^-$ event distribution in the $J/\Psi \to \phi\pi^+\pi^-$ decay. The width of the bin is 25 MeV. The solid line corresponds to the fit Eq.(44) with $q_{\max} = 0.9\,\mathrm{GeV}$. The dashed line is the best fit with $q_{\max} = 1\,\mathrm{GeV}$ and analogously the dashed–dotted line for $q_{\max} = 0.8\,\mathrm{GeV}$.

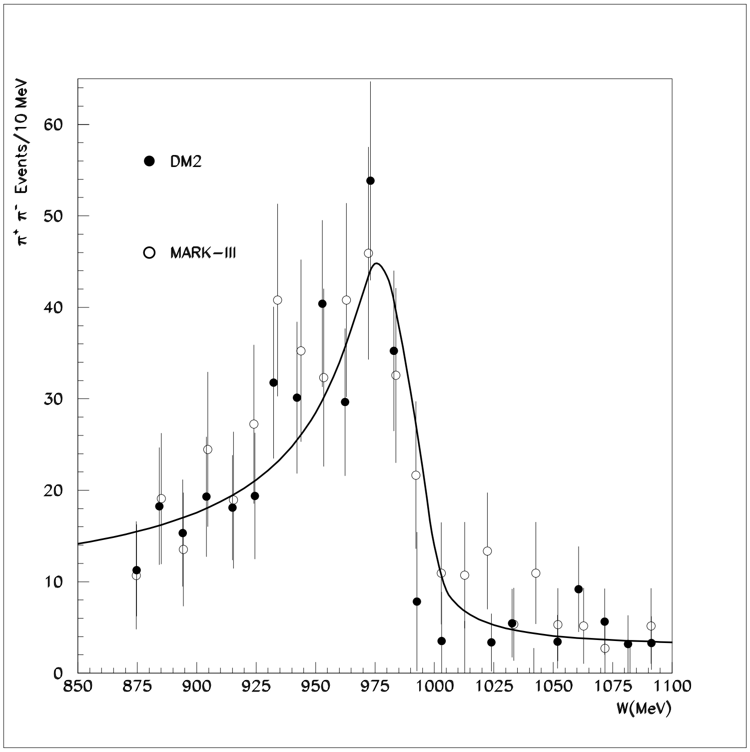

FIG. 8. $\pi^+\pi^-$ event distribution in the $J/\Psi \to \phi\pi^+\pi^-$ decay around the $f_0(980)$ mass. The width of the bin is 10 MeV.

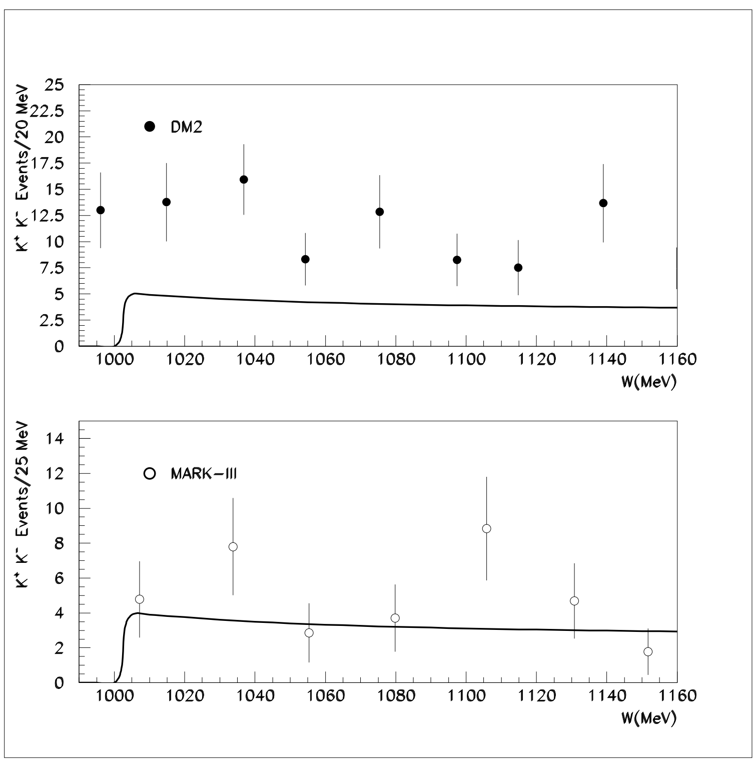

FIG. 9. K^+K^- event distribution in the $J/\Psi \to \phi K^+K^-$ decay. The upper panel corresponds to the data from DM2 [31], 20 MeV bins. The lower one corresponds to the data from MARK-III [35], 25 MeV bins.

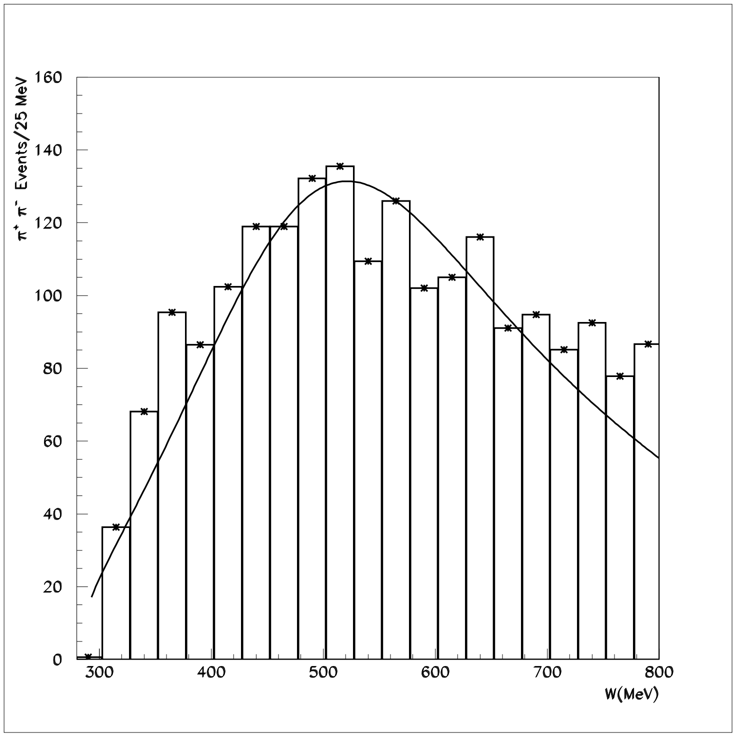

FIG. 10. $\pi^+\pi^-$ event distribution in the $J/\Psi \to \omega\pi^+\pi^-$ decay. The width of the bin is 25 MeV. Only the S-wave contribution is calculated. The onset of the D-wave contribution can be seen at energies larger than 700 MeV.